Analysis of Two Adaptive Filters in Tandem

Maurice Givens, *Senior Member, IEEE*

***Abstract*—An analysis of the error power of two adaptive filters in a tandem connection is presented. This connection configuration is often encountered using echo cancellers in the telecommunication network. This paper presents an analysis of the algorithm error power rather than the mean weight-error.**

***Index Terms*—Adaptive filters, adaptive systems, echo cancellers, NLMS algorithm**

I. Introduction

SINCE its introduction by Bernard Widrow, [1], adaptive systems have become widely used. Probably the most used adaptive system algorithm is the least-mean-square (LMS) algorithm. The LMS algorithm has been used for noise reduction, channel equalization, system identification, etc [2]. The use of low-cost high-speed components (e.g., DSPs, FPGAs) has made real-time coefficient adaptation possible for the most simple to the most complex adaptive systems.

One application in particular of adaptive systems is the now ubiquitous echo canceller. Figure 1 shows a typical telephone configuration using a 2-wire-to-4-wire hybrid and echo canceller. The hybrid forms an interface between the 4-wire switching system and the 2-wire telephone.

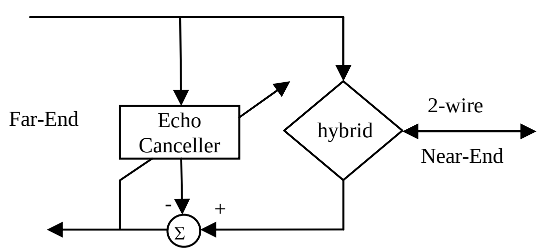

Figure 1. Typical Telephone Configuration With Echo Canceller

Since the hybrid is not a perfect circuit element, a portion of the signal from the far-end subscriber is returned to the subscriber, and is heard as echo when the delay exceeds approximately 35 ms. The echo canceller is employed in an adaptive system identification configuration. The echo canceller estimates the impulse response of the hybrid and generates an estimate of the echo which is then subtracted from the returning signal. This eliminates or minimizes the echo heard by the far-end subscriber. Often, echo cancellers can be placed in a tandem connection, usually when conference connections are made. The effect of tandem connections on echo canceller performance is of interest.

M. Givens is with Givens Burnet, Inc., Chicago, IL 60643 USA (phone: 708 601 8993; e-mail: maury001@core.com). At the time this work was performed, Mr. Givens was with Tellabs, Inc.

II. The Tandem Connection

When echo cancellers are placed in a tandem connection, the configuration of Figure 2 of occurs.

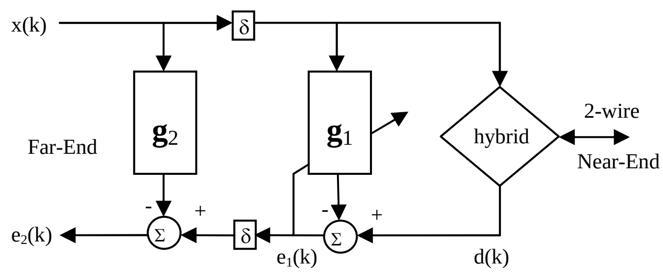

Figure 2. Tandem Echo Canceller Configuration

Previous analysis of tandem adaptive filters, [3], assumes a non-stationary unknown hybrid impulse response. Because of this assumption, the analysis uses weight-vector noise and weight-vector lag to determine the “misadjustment”, or excess mean-squared error, which is consistent with [4]. However, for echo cancellers, this non-stationary condition is a transient, not a steady-state condition. Consequently, the conclusion drawn by [3] that echo cancellers placed in tandem will experience an increase in the steady-state error of 4 dB is not consistent with an analysis of the system steady-state error power as opposed to the weight-vector error.

The delay in $e_1(k)$ caused by the delay δ, is integral to the update of $\mathbf{g}_2$, and should be modeled as such, instead of being incorporated into the input buffer of $\mathbf{g}_2$. Another factor to be considered is that because of δ, the input signal x(k) seen by $\mathbf{g}_1$ is a delayed version of that seen by $\mathbf{g}_2$.

A. Analysis of the Tandem Connection

The analysis is concerned with the behavior of adaptive filter $\boldsymbol{g}_2$, since it is clear that $\boldsymbol{g}_1$ follows accepted adaptive filter theory for exponential convergence [5]. The output of the system, $e_2(k)$ is

$$e_2(k) = \mathbf{h}^T \mathbf{x}(k-2\delta) - \mathbf{g}_1{}^T(k-\delta)\mathbf{x}(k-2\delta) - \mathbf{g}_2{}^T(k)\mathbf{x}(k) \quad (1)$$

bold variables indicate vectors, T indicates transpose, δ is time delay, and $\boldsymbol{h}^T$ is the impulse response of the hybrid. It is clear that some information derived from an analysis of $\boldsymbol{g}_1$ is needed. Define a weight-error vector $\boldsymbol{c}(k)$ as a translation of the principal coordinate system such that [2]

$$\mathbf{c}(k) = \mathbf{g}_1(k) - \mathbf{h} \qquad (2)$$

If $\boldsymbol{g}_1(k)$ is updated using the LMS algorithm

$$\mathbf{g}_1(k+1) = \mathbf{g}_1(k) - 2\mu e(k)\mathbf{x}(k) \qquad (3)$$

then $\boldsymbol{c}(k)$ can be found using the recursive algorithm

$$\mathbf{c}(k+1) = [\mathrm{I} - \mu\mathrm{R}]\mathbf{c}(k) \qquad (4)$$

I is the diagonal identity matrix and **R** is the auto-correlation matrix of the input vector, $\boldsymbol{x}(k)$.

The unitary similarity transformation [6] is used to express the auto-correlation matrix of the input, $\mathbf{R} = \mathbf{Q}^T \Lambda \mathbf{Q}$, and defining a rotation of the principal coordinate system [2] as $\boldsymbol{v}(k) = \mathbf{Q}^T \boldsymbol{c}(k)$, variable $\boldsymbol{v}(k)$ can be written as

$$\mathbf{v}_1(k+1) = [\mathrm{I} - \mu\Lambda]^k \mathbf{v}(0) \qquad (5)$$

Λ is a diagonal matrix of the eigenvalues of **R**, and **Q** is a matrix whose columns are the eigenvectors of **R**, $\boldsymbol{v}(0) = -\mathbf{Q}^T \boldsymbol{h}$, and the subscript 1 denotes the association of $\boldsymbol{v}(k)$ to filter $\boldsymbol{g}_1(k)$.

With regard to the adaptive filter $\boldsymbol{g}_2(k)$, the hybrid, $\boldsymbol{g}_1(k)$ and the delays are a time-varying unknown system in a system identification problem. For $\boldsymbol{g}_2(k)$, the system configuration is

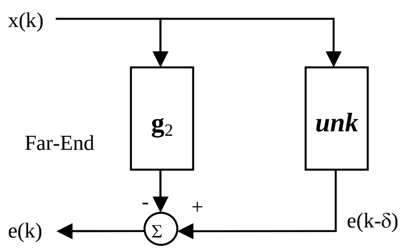

Figure 3. Tandem Adaptive Filter Configuration

and the error output signal is

$$e(k) = \mathbf{h}^T \mathbf{x}(k-2\delta) - \mathbf{g}_1{}^T(k-\delta)\mathbf{x}(k-2\delta) - \mathbf{g}_2{}^T(k)\mathbf{x}(k) \qquad (6)$$

with $e_1(k-\delta) = \boldsymbol{h}^T\boldsymbol{x}(k-2\delta) - \boldsymbol{g}_1^T(k-\delta)\,\boldsymbol{x}(k-2\delta)$ and $\boldsymbol{unk} = \boldsymbol{g}_1(k-\delta) - \boldsymbol{h}$.

The weight-error vector is $\boldsymbol{g}_2^T(k) - \boldsymbol{unk}$, or $\boldsymbol{c}_2(k) = \boldsymbol{g}_2^T(k) - (\boldsymbol{g}_1(k-2\delta) - \boldsymbol{h})$. As was done in (4), the weight-error vector is written as the recursive equation

$$\mathbf{c}_2(k+1) = [\mathrm{I} - \mu\mathrm{R}][\mathbf{g}_2(k) - (\mathbf{g}_1(k-2\delta) - \mathbf{h})]; k \ge 2\delta \qquad (7)$$

This indicates that the weight-error vector is $\boldsymbol{0}$, the null vector, until $k = 2\delta$. Again using the unitary similarity transformation $\mathbf{R} = \mathbf{Q}^T \Lambda \mathbf{Q}$, (7) is rewritten as

$$\mathbf{v}_2(k+1) = [\mathrm{I} - \mu\Lambda][\mathrm{Q}^T\mathbf{g}_2(k) - \mathrm{Q}^T(\mathbf{g}_1(k-2\delta) - \mathbf{h})] \qquad (8)$$

until $k = 2d$, $\boldsymbol{v}_2(k) = \boldsymbol{v}_2(0) = -\boldsymbol{v}_1(0) = \mathbf{Q}^T\boldsymbol{h}$, and $\mathbf{Q}^T(\boldsymbol{g}_1(k-2\delta) - \boldsymbol{h}) = \boldsymbol{v}_1(k-2\delta)$, and consequently

$$\mathbf{v}_2(k+1) = [\mathrm{I} - \mu\Lambda]^k \mathrm{Q}^T\mathbf{h} - \mathbf{v}_1(k-2\delta) \qquad (9)$$

What is of interest is the error power (residual echo) rather than the mean weight-error. The mean-squared error is determined from [7]

$$J(k) = J_{\min} + \sum_{n=1}^{M} \lambda_{2n} \left| v_{2n}(k) \right|^2 \qquad (10)$$

$J_{\min}$ is the minimum mean-squared error produced by the optimum Weiner filter, v_{2n} is the nth natural mode for $\boldsymbol{v}_2(k)$, and λ_{2n} is the eigenvalue corresponding to $\boldsymbol{v}_{2n}$

III. Simulations

Testing and simulations of the tandem connection were conducted. For the simulations, tests and analysis, white noise (Figures 4, 5, 6, and 7) and speech (Figures 8, 9, and 10) where used as inputs. The hybrid was modeled using m4 of G.168.

In Figure 4, a comparison between the analytical results and simulation is shown.

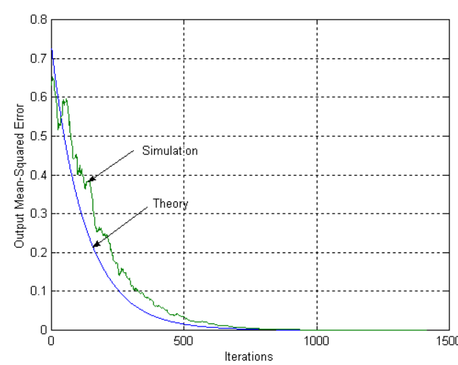

Figure 4. Comparison of Simulation and Theory

This comparison shows that the analysis and the simulations are in agreement. Further testing with actual echo cancellers was conducted to examine agreement between actual echo canceller implementation and the analysis.

A test was conducted with actual echo cancellers to examine the difference between the tandem and non-tandem output levels. The result of the test is shown in Figure 5.

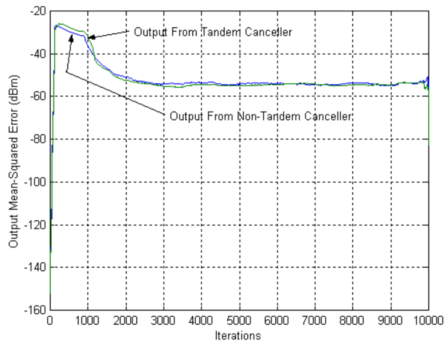

Figure 5. Comparison of Tandem and Non-tandem Cancellers

This shows the difference in convergence speed and the steady-state output level difference. An expanded view of this output is given in Figure 6. This shows a difference of approximately 1.0 dB between the tandem and non-tandem echo cancellers.

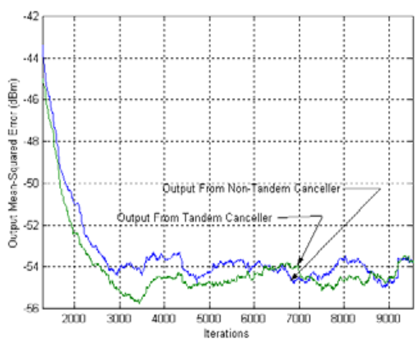

Figure 6. Comparison of Tandem and Non-tandem Cancellers

The analysis (Figure 7) shows a similar difference of 1.0 dB between the tandem and non-tandem echo cancellers.

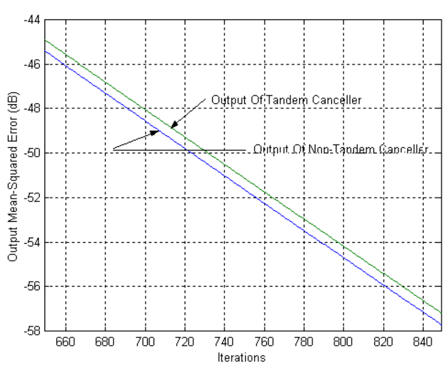

Figure 7.Comparison of Tandem and Non-tandem Cancellers Analysis

A speech signal, Figure 8, was used as a non-stationary input signal for simulations.

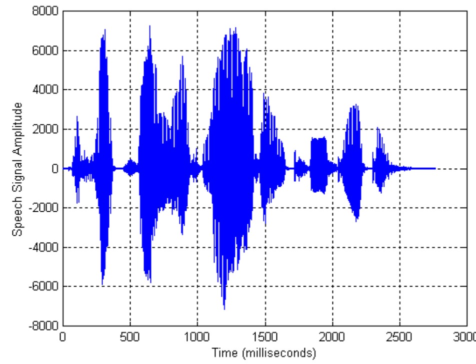

Figure 8. Speech Input Signal

The output mean-squared error for this non-stationary signal is given in Figure 9.

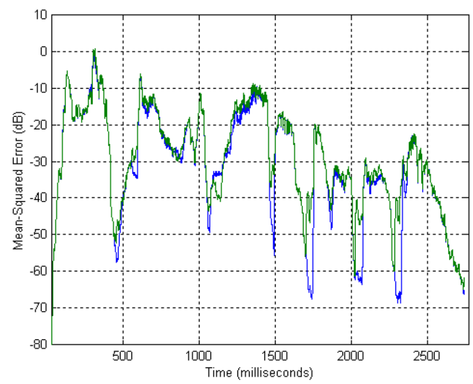

Figure 9. Simulation Output Mean-Squared Error (dB)

An increase in the output mean-squared error can be seen when the speech input changes characteristics (e.g., between 1500 and 2000 ms). However, once the non-tandem echo canceller adapts to the changed characteristics, the difference between the tandem and non-tandem echo cancellers is not appreciable. This is more clearly shown in the expanded view of this area in Figure 10.

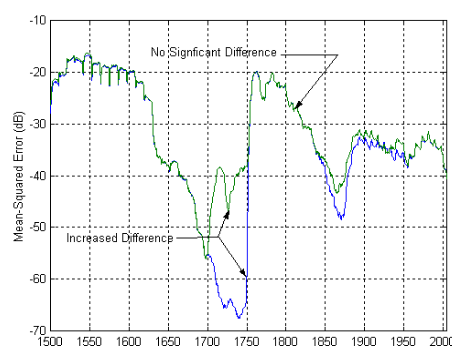

Figure 10. Expanded View of Output Mean-Squared Error

IV. Conclusion

Using the adaptive system configuration of Figure 2 and the accompanying assumptions, the analysis infers:

1. The time to converge to steady-state is increased when adaptive filters are placed in a tandem configuration.
2. During initial convergence, the residual echo (system error) is increased.
3. The steady-state residual echo (system error) is not appreciably increased (1dB) when adaptive filters are placed in a tandem configuration.

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